Assignment 10

Coverage: 16.4 and 16.5(part) in Text.

Exercises: 16.3 no 33, 38; 16.4 no 7, 11, 14, 23, 26, 28, 35, 37.

Hand in 16.3 no 33, 16.4 no 11, 14, 28, 35 by March 29.

Supplementary Problems

1. Verify Green's theorem when the region D is the rectangle $[0,a] \times [0,b]$.

2. Let D be the parallelogram formed by the lines $x+y=1, x+y=3, y=2x-3, y=2x+2$. Evaluate the line integral
$$\oint_C dx + 3xy\,dy$$
where C is the boundary of D oriented in anticlockwise direction. Suggestion: Try Green's theorem and then apply change of variables formula.

3. Let $F = M\mathbf{i} + N\mathbf{j}$ be a smooth vector field which is defined in $\mathbb{R}^2$ except at the origin. Suppose that it satisfies the component test $M_y = N_x$. Show that for any simple closed curve γ enclosing the origin and oriented in positive direction, one has
$$\oint_\gamma M dx + N dy = \varepsilon \int_0^{2\pi} \left[-M(\varepsilon\cos\theta, \varepsilon\sin\theta)\sin\theta + N(\varepsilon\cos\theta, \varepsilon\sin\theta)\cos\theta\right] d\theta \ ,$$
for all sufficiently small ε. What happens when γ does not enclose the origin?